Frege and the Logical Notion of Judgment

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0 Introduction

In this paper, I investigate a tension in Frege's mature philosophy arising from, on the one hand, a commitment to sharply distinguish the logical from the psychological, and, on the other hand, the introduction into his logical notation of a symbol that is meant to express the act of judgment. In response to this potential tension, I'll introduce a distinction from James Conant (forthcoming) between "Leibnizian" and "Kantian" conceptions of logic, and show how it is possible to arrive at two radically different ways of thinking about Frege's logic that each fall under one of these respective headings, one by the way he gives sense to his logical notation in his mature philosophy in terms of truth and falsity, and one suggested by the way in which he gives sense to his logical notation in the *Begriffsschrift* (1879) in terms of affirmation and denial. I claim that, while his mature conception does indeed have a serious tension when it comes to the role of the judgment stroke, there were the seeds of a very different conception of logic in his early writing, never developed by Frege, in which the judgment stroke is not at all out of place. After fleshing out this conception a bit, drawing from some neo-Aristotelian ideas of Irad Kimhi (2018), I'll close with a consideration of what logical necessity, on such a conception, can be.

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1   A Puzzle about Judgment

In the introduction to the *Foundations of Arithmetic*, Frege states that one of the basic principles to which he has aimed to adhere throughout the work is "always to separate sharply the psychological from the logical, the subjective from the objective," (1888/1960, xxii). The importance of this principle in Frege's work cannot be overstated. In order to understand what Frege means when he uses any of his logical expressions, we need to keep his distinction between the psychological and the logical clearly in view. The most famous discussion of this point comes in *Concept and Object* (1892/1997), where Frege explicates the logical rather than psychological way in which he uses the expression "concept," but this point applies equally well to Frege's use of the expression "thought." The expression "thought," like the expression "concept," can be used in various ways, some logical and some psychological, and when Frege uses the expression, he means to use it in a strictly logical rather than psychological sense, an objective sense rather than a subjective one.

We can begin to get the distinction between the objective and subjective senses of "thought" into view by considering the distinction between the things we think and our acts of thinking these things. We might start by explicitly noting that you and I can both think the same thing. For instance, we can both think that the Earth revolves around the Sun. That is, the thought that the Earth revolves around the Sun is something that we can both think. If we do both think this thought, we will have each thought it in our respective acts of thinking. Now, these acts must be distinct. After all, my act of thinking is *mine*, and your act of thinking is *yours*. So, there is a distinction we can make here between the one thought that we both think and our two acts of thinking it. It's

natural to conclude from this, that, when Frege uses the expression "thought," he uses it only to speak of things of the former sort. That is, he uses the expression "thought" in the objective sense, using it to speak of the objective things that we are able to think rather than our subjective acts of thinking those things. To use it in this latter sense, using it to speak of "acts of an individual mind" (1888/1997, 90), would be to use it psychologically, and Frege does not use it that way.

It might now seem as if we have clear-cut distinction between a logical and psychological sense of the expression "thought:" when "thought" used in its logical sense, it is used to speak of the things that we can think, and, when it's used in its psychological sense, it's used to speak of our acts of thinking such things. Things, however, aren't as simple as they might initially seem. Importantly, not only can we make a distinction between thoughts and thinkings, but we can also make a distinction between two kinds of thinkings, acts of entertaining a thought as something that might be the case and the acts of judging that a thought actually is the case. So, there are two types of acts here, acts of entertainment and acts of judgment. Now, it is clear that Frege regards the former type of act as psychological and subjective.[1] However, his view of the latter type of act is not so clear. Here's why: Not only does Frege employ the natural language expression "judgment" for this type of act, but he also employs a very different kind of expression for this kind of act, an expression that does not have its home in any natural language, but, rather, belongs to a purely formal one: a language that Frege calls "the *Begriffsschrift*."

The *Begriffsschift* is, as Frege describes it, "a formula language for pure thought." In this language, we express a judgment as follows:

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[1]Is it so clear?

$$\vdash\!\!\!-\!\!\!- A$$

This expresses the judgment of *A*, the act of recognizing *A* as true (329). Frege takes it that this judgment can be articulated into two parts, the thought that is recognized as true and the recognition of the truth of it. He writes, "when something is judged to be the case, we can always cull out the thought that is recognized as true; the act of judgment forms no part of this," (1915/1997, 322). That is what Frege's use of the complex symbol

$$\vdash\!\!\!-\!\!\!-$$

is meant to capture. We can consider this symbol as consisting in two parts, a horizontal line and a vertical line. Frege calls the horizontal line the "content stroke" and he calls the vertical line the "judgment stroke." In this way, the notation allows the content that is judged to be isolated from the act of judgment. If we want to express the content that is judged in the judgment that *A* apart from the judgment of it, we can do so as follows:

$$-\!\!\!-\!\!\!- A$$

Once we have such an expression of judgeable content, if we then want to express the judgment of it, we can add the vertical stroke to it, thereby giving us

$$\vdash\!\!\!-\!\!\!- A$$

which expresses the judgment of *A*.

Here, now, we have a puzzle. Frege vows "always to separate sharply the psychological from the logical, the subjective from the objective," (1888/1960, xxii). He articulates a conceptual line here. Below the line, he places the psychological and the subjective. Above the line, he places the logical and the objective.

Now, on our first pass at the distinction between the psychological and logical senses of "thought," we placed our acts of thinking below the line and the things that we think above it. This seemed to be in accord with Frege's association of "acts of an individual mind" with the psychological. However, in Frege's official logical notation, we have a particular expression, the vertical stroke, that is meant to express the act of judgment. If we use Frege's notation to articulate the laws of logic, we will, and indeed must, use this stroke. This suggests that Frege must be employing some notion of judgment here that belongs to *logic* rather than *psychology*—a logical, rather than psychological, notion of judgment. What could such a notion be? If judgment is a particular type of mental act and acts of an individual mind fall below the line dividing the psychological from the logical, to whom could the acts of judgment, expressed in Frege's logical notation, belong? Presumably, they could not belong to you, me, or Frege, since then they would be acts of an individual mind. So, to whom do these acts belong?

Some commentators take it that Frege cannot offer an adequate answer to this question. The judgment stroke, on these readings, manifests subjective acts of judgment, and, as such, is a violation of Frege's principle to separate the logical from the psychological. In his recent book *Thinking and Being*, Irad Kimhi (2018) writes,

> [The commitment] "to separate sharply the psychological from the logical" is an injunction against confusing anything manifested by the logical symbolism with subjective episodes or states (ideas, sensations, etc.). But Frege violates this injunction by introducing the judgment/assertion stroke into his logic, since its role is to manifest the actual act of assertion and so it is external to the significant functional composition of a proposition, (46).

Here, Kimhi implies that what is manifested by the judgment stroke—the actual

act of assertion—can be nothing other than a "subjective episode," and, as such, the inclusion of the judgment stroke in the logical notation is a violation of the principle to separate the psychological from the logical. Many people find more flatfooted variant of this criticism of the judgment stroke in found Wittgenstein's *Tractataus*:

> Frege's assertion sign "⊢" is logically altogether meaningless; in Frege (and Russell) it only shows that these authors hold as true the propositions marked in this way, (1922, §4.442, 52).[2]

Clearly, if all that the attachment of the judgment stroke to some proposition does is show that the author holds the proposition to which it is attached to be true, then, by Frege's own criteria, it does not belong to logic. Its function would be to manifest an act of an individual mind, that of the author. But the fact that a particular author holds a proposition to be true is a mere psychological fact about that author, and should have no place in logic. So, if this is what the judgement stroke is functioning to do in Frege's logical notation, it does not belong there.

It seems clear, however, that Frege's judgment stroke is at least not *meant* to

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[2]There is a debate to be had in the exegesis of Wittgenstein over exactly what Wittgenstein means here in the *Tractatus* in saying that the judgment stroke is "logically altogether meaningless." One question to be asked is whether Wittgenstein means the same thing here as he does in the *Investigations* when he characterizes the judgmment stroke as "superfluous," (1953, §22). Gustaffson (2017) gives a reading in which he means essentially the same thing, and Conant (forthcoming, and from personal conversation) construes the issue similarly. According to this sort of reading, Wittgenstein's remark about the judgment stroke in §4.442 does not indicate that Wittgenstein does not think that the notion of judgment does not belong to logical identity of the proposition, but, rather, indicates that Wittgenstein thinks that the notion of judgment is so fundamental to the logical identity of the proposition that it is a mistake to represent the act of judgment in the logical notation as something that is *added to* the proposition; the force of judgment must to already be *internal to* the proposition in order for the proposition to have the logical identity that it has. Accordingly, the judgment stoke, in construing the force of judgment as something that is added to the proposition, is "meaningless" in the sense of being superfluous, trying to add something to the proposition that must already be contained within it. Accordingly, what he says here in the *Tractatus* and what he says in the *Investigations* can be seen as the expression of the same basic point.

show that the individual author holds the thoughts to which it is prefixed to be true. After all, the thought that Frege judges A to be true might be expressed in the *Begriffsschrift* as $J(f, a)$, where $J(x, y)$ is a function whose value *true* just in case x judges y, f is Frege, and a is A.[3] This is a perfectly legitmate expression of Begriffschift, and, if an empirical psychologist was conducting a study on Frege and using Begriffsschrift to be particularly precise in drawing conclusions, this may well be something they write down in the course of their study. But it is not something that a *logician* would write down. The logician is not concerned with subjective episodes of individual minds in this way. When a particular logician, Frege, say, expresses the judgment that A in *Begriffsschrift*, this judgment is not expressed in this way. As we've said, it's expressed like this:

$$\vdash\!\!\!-\!\!-A$$

Here, unlike the predicate letter J in the formula $J(f, a)$, the judgment stroke does not express anything that constitutes a part of the *content* that is expressed here. The judgment stroke is not part of but stands *outside* the content of which it expresses the judgment.[4] Accordingly, what is expressed by the judgment stroke here here cannot be the same thing or even the same sort of thing as what is expressed by the predicate letter "J" in the formula $J(f, a)$. Perhaps more strikingly, there is no reference to any individual who is making the judgment at all; there is no expression that is meant to refer to any individual judger, be it Frege, the reader, or anyone else. Given that a fundamental aim of the

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[3]Mark Textor (2010) claims that Frege's notion of judgment is better construed as a 3-place relation in which the truth value that the subject judges a thought to have also occupies of a place of the relation. While Textor is probably correct that *if* judgment is to be construed as a relation, it should be construed as a 3-place relation, I ignore this consideration for simplicity, since I take it to be irrelevant to the point that the logical notion of judgment is not to be construed as a relation at all.

[4]Kimhi (2018) thus describes it as the "sole syncategorematic expression" of the *Begriffsschrift* (41).

Begriffsschrift is to "reveal every presupposition that tends to slip in unnoticed" in everyday language (1879/1997, 49), it would be quite strange if Frege just left a reference to himself or some other judger implicit in the notation. Surely not. Rather, it seems clear that he's working with a fundamentally different notion of judgment here than the notion of judgment that could be expressed with a formula like $J(f, a)$: a logical rather than psychological notion of judgment.

In a recent paper, Maria van der Schaar (2017) suggests that we understand these two notions of judgment not in terms of two distinct types of judgment, but, rather, in terms two perspectives that one might take on judgment. She writes:

> [Judgment] may be looked at from two different perspectives: as a phenomenon in the empirical world, to be studied by psychology, and as a logical phenomenon, standing under objective logical norms constituted by the norm of truth and the laws of truth, (238).

Understood from the third-personal perspective, a judgment is an empirical happening in the world, an episode of a particular individual to be studied by empirical psychology. Understood from the first-personal perspective, however, to judge something is to take it to be correct to judge, to take one's act of judging it to be in conformity with the norms that govern what is to be judged, most fundamentally the norm that one is to judge only what is true and the norm that one is not to judge in such a way that one's judgment cannot possibly be true. To express an act of judgment from this perspective is not to make an empirical report on one's own subjective episode, but to express an act of judgment as conforming to the norms governing what is to be judged, for it is just in taking the judgment to be in conformity with these norms that one makes the judgment. Accordingly, insofar as acts of judgment expressed in

Begriffsschrift are understood from this first-personal perspective, in response to the question "to whom do these acts belong?" one may simply respond: someone whose acts of judgments are governed by the norms governing what is to be judged. Insofar as one is expressing one's judgment from this first-personal perspective, any reference to oneself as the particular judger that one is is irrelevant. As van der Schaar says, "These judgements are thus made from a first-person perspective, but they are non-personal at the same time," (240) and this is what the notation for expressing judgment, in lacking any place for reference to an individual judger, makes explicit.

2 A Conception of Logic in Which Judgment Has No Place

Is the puzzle about the judgment stroke so easily solved? In his forthcoming work *The Logical Alien*, James Conant suggests that it is not.[5] In the work, Conant draws the same distinction that van der Schaar draws between the psychological and logical notion of judgment in Frege's thinking.[6] Elaborating on what the logical notion of judgment must be, for Frege, Conant goes on to

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[5]I pursue Conant's way of thinking that the problem is not so easily solved here, but is worth noting that a more direct way of objecting to van der Schaar's particular construal of the logical notion of judgment that I will not consider here can be found in Sebastian Rödl's *Self-Consciousness and Objectivity*. There, Rödl objects to the construal of the science of judgment as a science from "the first-person perspective," claiming that "The self-consciousnesses of judgment is not a perspective *on* judgment. *It is judgment*," (60). This suggest that, in fact, we are equivocating on the use of the term "judgment" when we speak of "judgment," as it is studied in empirical psychology and "judgment" as it is studied in logic. It is worth investigating how this claim of Rödl's meshes with Conant's own way of construing the distinction between logic and empirical psychology.

[6]Conant draws the distinction between the two notions of judgment as follows:

> The term "judgment" in Frege's hands [. . .] can denote either a properly logical or a merely psychological notion. The logical notion of judgment pertains to the acknowledgment of those truths that ought to be asserted and whose logical structure is to be rendered apparent in Frege's Begriffsschrift. [. . .] The psychological notion of judgment, on the other hand, pertains merely to actual episodes of particular judging subjects holding particular judgments to be true, (forthcoming, 143).

say the following:

> The presence of the judgment stroke is to be understood as indicating that the content that it encloses is taken up in a general act of mind. Thus Frege requires a conception of the power of judgment (whose exercise the judgment stroke seeks to denote) in accordance with which it is at one and the same time general and actual. It must not be taken to indicate some empirical individual's exercise of the power but [...] rather a generic instance of the exercise of a general power, where our very conception of the power depends not only upon our conceiving the actuality of its exercise as a condition of its reality, but upon our actually exercising it, (forthcoming, 360).

Though Conant says here that this is how Frege's judgment stroke is to be understand, he also claims that Frege himself (unlike, say, Kant) does not have a conception of logic that enables him to spell out the notion of a "generic instance of the exercise of a general power" that making sense of the inclusion of the judgment stroke in his logical notation requires.[7] Conant ties this claim to Frege's dissociation of the act of judgment from the content that is judged. This dissociation leads Frege to think logic as principally concerned with relations among thinkable contents which are taken to be intelligible apart from any actual exercise of the power of thinking. Logic thus concerns what is thinkable rather than the power of thinking, and, on such a conception, the judgment stroke is out of place. Let me spell out this claim, as I understand it.

In Frege's mature philosophy, logic is concerned, in the first instance, not with acts of thinking, but with thoughts, where "thought" is understood here as denoting an objective thing that is thinkable rather than a subjective act of thinking such a thing. A thought, Frege says, is "something for which the question of truth can arise at all," (1918/1997, 328). Elaborating a bit, a thought

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[7]Indeed, Conant goes as far as Frege could only say what is meant by such a notion with "embarrasment," (360).

is something whose truth turns on whether or not things are a certain way. For instance, the thought that Earth revolves around the Sun is a thought whose truth turns on whether or not the Earth revolves around the Sun. If the Earth revolves around the Sun, the thought is true; if not, the thought is false. All sciences are concerned with thoughts. Astronomy, for instance, is concerned with the thought that the Earth revolves around the Sun; it concerns whether this thought is true, it concerns the relationship that this thought bears to other thoughts, for instance, the thought that Earth is a planet, or the thought that all the planets revolve around the Sun, and it also concerns whether these other thoughts are true. So, Astronomy concerns a particularity class of thoughts, those whose truth turns on astronomical matters. History concerns a different class of thoughts, music theory concerns yet another class of thoughts, and so on, for all the other sciences—all the other sciences, that is, except logic. Unlike astronomy, history, or music theory, which concern a particular class of thoughts—those whose truth turns on astronomical, historical, musical matters—logic concerns *all* thoughts. So, logic concerns the thought that the Earth revolves the Sun, the thought that John Wilkes Booth shot Abraham Lincoln, the thought that C lies three semitones above A, and so on. It concerns all of these thoughts insofar as they are all thoughts of the form that some thing a stands in some relation R to some other thing b. Logic concerns relations any of these thoughts, considered at this level of generality, bear to other thoughts, also considered at this level of generality. One such relation is that if some thing, a, is some way, F, and every thing, x, is such that, if x is F then x stands in some relation, R, to some thing, b, then a stands in R to b. This is a bit of a mouthful in English, but, in Frege's notation, we'd express this thought like this:[8]

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[8]In modern notation we would write this thought as one of the following logically equivalent
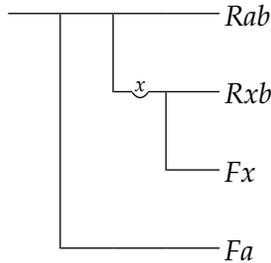
This complex symbol expresses a relation that obtains between thoughts about celestial bodies, thoughts about historical figures, thoughts about music notes, or whatever else about which we might think. In this way, logic concerns the relations that thoughts stand to one another not in virtue of being about the particular things that they are about or representing these things as being the particular ways that they represent them as being, but simply in virtue of being about *things* and representing these things as being certain *ways*.[9] In concerning itself with thoughts at the level of generality that it does, then, logic concerns itself with every thing that can be thought about and every way that any thing that can be thought about can thought to be.

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formulas:

$((Fa \land \forall x(Fx \supset Rxb)) \supset Rab)$

$(Fa \supset (\forall x(Fx \supset Rxb) \supset Rab))$

MacBeth (2005) argues that the fact that Frege's two-dimensional notation actually can be read in either way is a notable feature of the notation "one [two-dimensional] formula corresponds to an equivalence class of formulae in standard one-dimensional notation," (52). Unrelatedly, I should note that the types of letters that I'm using here and in Frege's notation are the more familar ones from our modern notation, not the ones that Frege himself uses.

[9]Talk of a thought's "representing" things as being certain ways is not particularly germane to Frege's own way of speaking, but it is hard to say exactly how it is that thoughts have truth conditions without employing some talk of this sort. Ultimately, I take it that there is no way of making sense of how Fregean thoughts, understood as mind-independent entities, can indeed have truth conditions (see Simonelli (MS) on this point), but I am trying to do my best here to speak as if the conception of thoughts in Frege is coherent, and I can only make sense of how a thought could have truth conditions by thinking of a thought as representing some thing as being some way or representing some set of things as standing in some relation.

This conception of logic endorsed in Frege's mature philosophy is a version of what Conant calls a "Leibnizian" conception of logic (forthcoming, 107-111). For Leibniz, there a class of modal truths that God is able to comprehend in virtue of possessing the capacity for understanding that He has, and, in virtue of comprehending these truths, He is able to survey the space of possibilities for the world independently of choosing to actualize one of them (the best one). Likewise, for Frege, there a class of logical truths that we are able to comprehend in virtue of possessing the capacity for logical understanding that we have, and, in virtue of comprehending these of truths, we are able to survey the space of possibilities for how things could logically possibly be independently of our actually judging that any particular thing actually is any particular way. It is not my aim here to criticize a Leibnizian conception of logic.[10] The point here is just that, if one has a Leibnizian conception of logic, as Frege does, it is extremely strange that one would have an expression for the act of judgment belonging to the language for expressing logical truths, as Frege also does.

On a Leibnizian conception of logic, logic's primary concern is not with the activity of thinking, but with the things that can be thought, the things that can be thought about, and the ways that these things can be thought to be. Logic is concerned with the activity of thinking only derivatively. If one aims to think truly, then, regardless of what it is about which one is thinking, one

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[10]While Conant is clearly critical of the Leibnizian conception of logic, I don't think it's a perfectly apt thing to say that one of his principle aims in the book is to argue that the Leibnizian conception of logic is wrong. It seems to me that Conant's primary aim, in his discussions of the Leibnizian conception and its alternatives, is to transform our understanding of the space of possibilities for thinking about logic. Once the conceptual transformation has taken place, once the new possibilities have made it onto the scene, Conant has done the thing that he has aimed to do. Saying, after articulating the space of possibilities, "And *this* possibility is the right one; the others are wrong" is not really part of his task. The point of the exercise is not to say that some conceptual possibility is the right one and the others are wrong, but to transform our understanding of what the conceptual possibilities are.

ought to think in such a way that one's acts of thinking conform to the truths articulated by logic. If one's thinking failed to conform to these truths, one would be thinking in such a way that, regardless of what it is about which one is thinking, one is thinking falsely. So, insofar as, when one judges, one aims to judge truly, the truths articulated by logic can not only be spoken of as "laws of truth" (1918/1997, 326), but also as "laws of judgment," (1897/1997, 246). However, in using this latter phrase, Frege warns us, "we must not forget that we are concerned here with laws which, like the principles of morals or the laws of the state, prescribe how we are to act, and do not, like the laws of nature, define the actual course of events," (1897/1997, 246-247). Whereas the laws of Newtonian physics are constitutive of how Newtonian bodies actually do act, the truths articulated by logic constitute "laws" for judgment in the sense that they prescribe how we ought to judge; they are in no sense constitutive of how we actually do judge. They are normative rather than constitutive laws.[11] Logic can be normative for judgment only insofar as we take ourselves to be bound by the norms determined by the logical truths. Accordingly, logic is normative for judgment only insofar as we already comprehend what the logical truths

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[11]I take it that this is how Frege thinks of the laws of logic, but it is not clear how well this distinction really holds up to scrutiny. John MacFarlane (2002), for instance, speaks of the laws of logic as "constitutive norms" for thinking as such. He compares this with the way in which the laws of physics "provide constitutive norms for the activity of thinking about the physical world. Only by opting out of that activity altogether—as one does when one is spinning a fantasy tale, for example, or talking about an alternative possible universe—can one evade the force of these norms," (37). In order to be "playing the game" of making judgments about the physical world, one must take oneself to be bound by the norms instituted by the laws of physics. If one is happy to judge in such a way that what one judges requires for its truth that things fail to accord with the laws of physics, it is hard to see how one could genuinely take oneself to be making judgments about the physical world all. Likewise, in order to be "playing the game" of judging at all, one must take oneself to be bound by the norms instituted by the laws of logic. If one is happy to judge in such a way that what one judges requires for its truth that things fail to accord with the laws of logic, it is hard to see how one could genuinely take oneself to be making judgments at all. If we follow this thought through, it is hard to see how these laws could not be constitutive of what thinking is, and, as such "define the actual course of events" insofar as these events are understood as genuine acts of thinking.

are and thereby comprehend how we are to judge if we are "not to fail of the truth," (1897/1997, 246). Only once the logical truths have been comprehended can we think of them as constituting laws for judgment so as to think of our acts of judgment as an acts "standing under objective logical norms constituted by the norm of truth and the laws of truth," (van der Schaar 2017, 238). Since the act of judgment, understood as an act governed by logical laws, can only come into the picture after the logical truths have been comprehended, and the topic of logic is these logical truths themselves, the act of judgment is external to the topic of logic.[12]

As a demonstration of this point, let us consider the actual expression of a judgment in Begriffschift notation. In order to have an actual logical truth that we are able to think about as such, and to which we can prefix the judgment stroke to consider its function, it will be helpful, for simplicity's sake, to consider a truth that obtains between thoughts in abstraction from any internal structure that they have, simply considered as things that are either true or false.[13] Given that every thought is either true or false, there are four and exactly four distinct possibilities for the truth or falsity of two given thoughts, *A* and *B*. They are the following:[14]

1. *A* is true and *B* is true.

2. *A* is true and *B* is false.
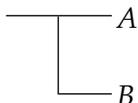
3. *A* is false and *B* is true.

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[12]Conant's way of putting this point is to say that, on a Leibnizian conception, "Logic articulates a body of truths whose relations of logical compatibility are prior to their compossibility in the consciousness of a finite thinking subject," (164).

[13]To think consider thoughts in this way is to ignore Frege's main logical innovations over contemporaries like Boole.

[14]Of course, on Frege's account, the sentence "*A* is true" is more perspicuously written as "*A* is the True." This has the same basic logical form as since what it is for a thought to be true is for that thought to pick out the True.

4. *A* is false and *B* is false.

Now, the complex symbol:

$$
\begin{array}{l} \rule{1cm}{0.4pt}\!\!\top\!\!\rule{0.5cm}{0.4pt}\ A \\[2pt] \qquad\ \rule{0.5cm}{0.4pt}\ B \end{array}
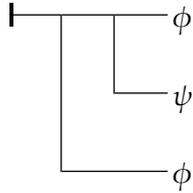$$

expresses the thought that is true just in case possibility (3) does not obtain, false just in case possibility (3) does obtain. That is, this complex symbol expresses the thought that is true just in case it is not the case that *B* is true and *A* is false.[15] The notation is thus simply defined in such a way that this complex symbol expresses the thought with exactly these truth conditions, which just is the thought that it is not the case that *B* is true and *A* is false. So, given the way the notation is defined, if this thought is true, then it is not possible for *B* to be true and *A* to be false. So, if this thought is true and *B* is true, then it is not possible for *A* to be false.

Now, let ϕ and ψ be any two thoughts—be they simple or complex and be they about celestial bodies, numbers, musical notes, or whatever else. Now consider the following formula:

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[15]In modern notation, this would be expressed as $(B \supset A)$ which is defined to be equivalent to $(\neg(B \wedge (\neg A)))$. The stroke itself that unites the symbols "*A*" and "*B*" expresses the function that takes a pair of truth values (α, β) and returns the True just in case it is not the case that α is the True and β is the False.

(1)

$$\vdash\!\!\!\begin{array}{l}\rule{1em}{0.4pt}\phi\\[2pt]\rule{1em}{0.4pt}\psi\\[2pt]\rule{1em}{0.4pt}\phi\end{array}$$

This formula, containing the judgment stroke, expresses a judgment. To see whether the content that this formula expresses the judgment of is a logical truth, consider what would have to be the case for the content that is judged to be false. The content that is judged in (1)—call it (1c)—is the following:
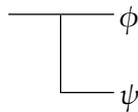
(1c)

$$\begin{array}{l}\rule{1em}{0.4pt}\phi\\[2pt]\rule{1em}{0.4pt}\psi\\[2pt]\rule{1em}{0.4pt}\phi\end{array}$$
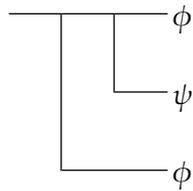
This content is false just in case

$$\rule{1em}{0.4pt}\,\phi$$

is true and

$$\begin{array}{l}\rule{1em}{0.4pt}\phi\\[2pt]\rule{1em}{0.4pt}\psi\end{array}$$

is false. Given how the notation has been defined, if the second content is false, then ψ is true and ϕ is false. Clearly, if ψ is true and ϕ is false, then ϕ is false. So, if the second content is false, then ϕ is false. But, if the first content is true, then

ϕ is true. Thus, if the first content is true and the second content is false, then ϕ is both true and false. But it is impossible for some content to be both true and false. So it is not possible for the first content to be true and the second content to be false. Since this is the only way for (1c) to be false, it is not possible for (1c) to be false. Thus, it is necessary that (1c) is true. So, as a matter of logic, we are entitled to make the following judgment:

(1)
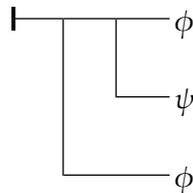
Formula (1) is a formula that we are able to write down in the course of doing logic. It is, as a matter of fact, the first formula that Frege writes down as an axiom in the *Begriffschifft*. Like all such formulas, it contains a judgment stroke. It expresses an act of judgment. Specifically, it expresses the judgment of (1c), a logical necesity. Note, however, that there is no univocal sense in which we can speak of the "necessity" of (1c) and the "necessity" of judging it. Frege understands the necessity of (1c) in terms of its absolute generality. Since every thought is either true or false, any two thoughts, regardless of what thoughts they are, stand in the relation that (1c) represents any two thoughts as standing in. (1c) is thus a necessary truth about thoughts in virtue of being a truth about every possible thought. If we understand "necessity" in this way, Frege is clear that, at least on logical grounds, there *is no* necessity involved in judging (1c). It is something that we "must" judge in the sense that we *ought* to judge it, but that doesn't mean that we *cannot but* judge it. Even if it does happen to

be a fact that we cannot but judge it, this is a fact that belongs to psychology, not to logic.[16] Any sense in which we can speak of our judgment of a logical truth as being "necessary" is not the sense in which the truth itself is necessary. Resolving to use "necessity" in the sense in which logical truths themselves are necessary, the act that (1) expresses is not necessary. Since logic's concern is with necessary truths, a formula that contains the judgment stroke does not express something that, properly speaking, belongs to logic; it only express the judgment of such a thing. The act of judgment—like the judgment stroke that expresses it, in standing outside of the content of which it expresses the judgment—stands outside of what, properly speaking, belongs to the topic of logic.[17] A thought just is or isn't a logical truth, and this has nothing to do with our judgment of it.[18]

3 A Conception of Logic in Which Judgment Has Pride of Place

Must we give up hope for a conception of Frege's logic in which the notion of judgment properly belongs to it? Perhaps not. There is, according to Conant, a live alternative to a Leibnizian conception of logic, what he calls a "Kantian" conception of logic. While Conant does not intend the phrase "Kantian conception

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[16]See Frege (1893/1997, 204).

[17]So why, then, does Frege, even in his mature writing, have the judgment stroke in his logical notation? There is, I take it, a simple pragmatic function that the judgment stroke plays. Just as I have done above with respect to (1), Frege, in the course of informally justifying his axioms (for instance, the rather complex axiom (2)), shows the reader unjudged contents, which are not logical truths, in order to informally reason about their possibilities for truth or falsity. The absence of the judgment stroke with respect these contents makes it clear that these contents are not to be judged but are just to be considered on the way to showing that the axiom, which is to be judged, is a logical truth. When Frege takes it that he has done enough to show that the only possibility that can obtain with respect to the axiom is the possibility of truth, he writes it down once again as a formula with a judgment stroke. The judgment stroke is a useful way of distinguishing to the reader which contents are presented as logical truths and which are not. The judgment stroke is thus pragmatically useful, but, as Wittgenstein says, it is "logically altogether meaningless."

[18]Though our judgment of it better have something to do with its being a logical truth.

of logic" to be synonomous with "Kant's conception of logic," it does not hurt to start with Kant in thinking about what such a conception might be. In the *Critique of Pure Reason*, Kant makes a distinction between two types of logic: transcendental logic and general logic. Transcendental logic, the new sort of logic that Kant develops in that work, aims to articulate the form of empirical cognition, the form of cognition insofar as it relates to an object other than itself. Cognition of an object, for Kant, is a joint act of the two basic faculties of mind, sensibility and the understanding. Sensibility is the faculty of mind through which an object is intuitively brought into view, and the understanding is the faculty of mind through which the object is conceptually grasped. Transcendental logic concerns the relation between sensibility and the understanding through which an object is cognized. General logic, on the other hand, "abstracts [...] from all content of cognition, i.e., from any relation of it to the object, and considers only the logical form in the relation of cognitions to one another, i.e., the form of thinking in general," (1781/1998, A55/B79, 195-196). That is, general logic abstracts from the way in which an object is given in sensibility and concerns itself with nothing but the core activity of the understanding—thinking. Our task in general logic is to articulate the form of coherent thinking. We articulate this for by articulating the "absoluterly necessary rules of thinking, without which no use of the understanding takes place," (A52/B76, 194). These are the rules according to which the understanding must proceed if it is to proceed coherently: in such a way that "it agrees with itself," (1800/1992, 527).

On a Kantian conception of logic, logic concerns itself with the activity of thinking in a way that, on a Leibnizian conception, it does not. On a Leibnizan conception, logic concerns itself with thinking in the sense that it concerns itself with what is thinkable, but it does not principally concern itself with the activity

20

of thinking itself. For Kant, by contrast, the activity of thinking is logic's proper concern. Logic articulates what it is for the capacity for thinking to be in act in such a way that one does not disagree with oneself. To make our terminology a bit more Fregean (and no less Kantian), logic, on this conception, concerns itself with the capacity of judgment. Logic articulates what it is for the capacity of judgment to be coherently in act—to be in act in such a way that it agrees with itself so that it can even possibly be in act in the recognition of some truth. That is, we are articulating the "form" of the capacity for judgment by articulating the form of its coherent activity in abstraction from any "matter" about which it is actually making judgments. In doing logic, what we are articulating are the ways for the the capacity for judgment to be in act such that, if it is in act in a way that is in opposition to any of these ways, it must be in disagreement with itself, and so, whatever the matter is about which it is judging, it cannot be judging sucessfully: its act of judging cannot be an act of knowing. If this is our conception of logic, then the notion of judgment will not be out of place at all. Furthermore, if we have an expression for the act of judgment in our logical notation, what such an expression will do is make explicit that what is being articulated by the logic notation are forms of the necessary activity of the capacity for judgment. The claim I'll now make is that, even though Frege's mature conception of logic is clearly Leibnizian, there were the seeds of alternate, Kantian conception of logic in his early way of giving sense to his logical notation. On this inchoate conception, the judgment stroke is not out of place at all.

In the above explication of Frege's mature conception of logic, I spoke of him considering the possibilities of truth and falsity of various formulas expressed in the notation of the *Begriffsschrift*. However, in the *Begriffsschrift*, when Frege

21

introduces the logical operators, he does not introduce them in terms of the possibilities of truth and falsity. He doesn't use the expressions "true" and "false" at all. Rather, he speaks of various contents being either "affirmed" or "denied." Most commentators regard Fege's early explication of his logical notation in terms of affirmation and denial as a mere terminological variant of his later, more sophisticated explication in terms of truth and falsity. Hans Sluga, for instance, writes "Frege uses the terms 'affirmed' and 'denied' in the *Begriffsschrift* in the sense of 'true' and 'false.' In his later writings the terminology is changed accordingly," (1980, 78). Michael Beany, in his commentary on the *Begriffsschrift* simply puts "i.e. are both true" next to Frege's statement of the possibility that two propositions are affirmed, (1997, 376). To construe this difference as a matter of mere terminology is, I think, misleading. It is, of course, a matter of terminology, but it would be wrong to say that it is a matter of *mere* terminology. What is at stake in this terminological decision is the core set of terms that we are going to employ to give sense to the logical notation. The terms we opt for will determine how we think about the logical notation that we use these terms to explicate. My claim is that, if we take Frege's early terminology seriously, we end up with a very different conception of Frege's logic, a Kantian rather than Leibnizian one.

Now, it's unclear whether Frege himself took his terminology particularly seriously at the time. Shortly after the publication of the *Begriffsschrift*, in "Boole's Logical Calculus and the Concept Script" (1881), he provides a brief overview of his logic in which "true" and "false" play the role "affirmed" and "denied" do in the *Begriffsschrift*. So, it seems that, at the time, he did not take himself to be particularly committed to the vocabulary of affirmation and denial for explicating his logical notation. He was aware that there were multiple sets

of expressions that were able to do the job and felt free to employ one set of expressions in the *Begriffsschrift* and another set of expressions elsewhere. Frege does not settle on an official semantic vocabulary for his logical notation until he develops the technical expressions "the True" and "the False," meant to denote what he calls "truth-values." Frege took the development of truth-values to enable him to provide a much cleaner explication of the meaning of his logical notation, and so this was a development from which he never returned. In the preface to the *Basic Laws of Arithmetic* writes,

> How much simpler and sharper everything becomes through the introduction of truth values, only thorough occupation with this book can show. These advantages alone already put a great weight on the balance in favor of my conception, which indeed may seem strange at first glance, (1893/1997, 198).

However, before this development, at the time in which the *Begriffsschrift* was written, Frege took the notions of affirmation and denial to be able to do the same work that he would later employ the notions of truth and falsity to do.

The idea that affirmation and denial have a central place in logic does not, of course, originate with Frege. Aristotle articulates an act of assertion as either consisting in a simple act of affirmation, a simple act of denial, and some complex of these two types (*DI* 5). Aristotle's syllogistic logic is based on the distinction between acts of affirmation and acts of denial. Thinking of logic in terms of relations among affirmations and denials continues in the *Port Royle Logic* (1662). Classical logic of this sort was was still the most pervasive conception of logic at Frege's day. So, Frege was not coming out of nowhere in explicating his logical system in terms of affirmation and denial; he was simply inheriting, as semantic primitives, the fundamental logical notions of the logic of the day. He would eventually leave these primitives behind, favoring the explication of his logical

23

notation in terms of truth values, but these are the primitives with which he started. What I want to suggest is that, in leaving the notions of affirmation and denial behind, Frege left behind a conception of what is articulated by his logical notation in which the judgment stroke has a proper place.[19]

We can get some grip on the sense of the expressions "affirm" and "deny" just by reflecting on our own acts of affirmation and denial in speech. If I ask you, "Is it the case that A?" you can affirm A, saying "Yes," or you can deny

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[19]Thinking about logic in terms of affirmation and denial has enjoyed a bit of a renaissance in recent years, particularly in the defense of Gentzen's (1936) proof systems for classical logic, of both the natural deduction and sequent variety, against objections that their intuitionistic counterparts are preferable.

In the case of the natural deduction systems, Dummett (1991) argues for an intuitionsitc understanding of the propositional connectives over a classical understanding of them on the grounds that the natural deduction rules for the intuitionistic connectives in Gentzen (1936) are harmonious (all of the rules conform to Prawitz's (1965) inversion principle), and so the meanings of the intuitionistic connectives can be understood in terms of the rules governing their use, but the rules for the classical connectives are not harmonious, and so their meanings cannot be so understood. However, Ian Rumfit (2000) shows that if we have a "bilateral" natural deduction system in which formulas are signed, positively or negatively, we can provide harmonious introduction and elimination rules for the classical connectives, especially negation. The bilateral rules for negation are the following, where "$+\phi$" signifies an act of affirming ϕ and "$-\phi$" signifies an act of denying ϕ:

$$\frac{-\phi}{+(\neg\phi)} \ +\neg_I \qquad\qquad\qquad\qquad \frac{+(\neg\phi)}{-\phi} \ +\neg_E$$

$$\frac{-(\neg\phi)}{+\phi} \ -\neg_E \qquad\qquad\qquad\qquad \frac{+\phi}{-(\neg\phi)} \ -\neg_I$$

These rules are clearly harmonious. Additionally, these rules can be seen as a codification an a natural deduction system of Frege's own way of thinking about negation, expressed when he says such things as "When it is a question of whether some thought is true, we are poised between opposite thoughts, and the same act which recognizes one of them as true recognizes the other as false," (1897/1979, 149). This bilateral rules codify this thought as follows: when it is the question of whether some thought ϕ is true, we are poised between two opposite thoughts, ϕ and $\neg\phi$, and the act of affirming ϕ is the same as (i.e. logically equivalent to) the act of denying $\neg\phi$ and the act of denying ϕ is the same as (i.e. logically equivalent to) the act of affirming $\neg\phi$.

In the case of sequent calculus, the issue is has to do with the fact that Gentzen's intuitionistic sequent calculus LJ contains single conclusion sequents, whereas the classical sequent calculus LK contains multiple conclusions. Whereas the antecedents of a sequent are to be read conjunctive, the conclusions are to be read disjunctively. Greg Restall (2005), however, shows that we if we think of the meanings of sequents in bilateralist terms, we can naturally make sense of multiple conclusion sequents. $\Gamma \vdash \Delta$ is to be read as asserting that affirming all the elements of Γ and denying all the elements of Δ is "out of bounds."

A, saying "No." You cannot coherently affirm *A* and deny *A*, saying both "Yes" and "No," at least not without doing so at two different times or specifying two different ways in which the question might be taken. Assuming that the question "Is it the case that *A*?" is taken at a single time in a single way, if you affirm *A*, you cannot coherently deny *A*, and if you deny *A*, you cannot coherently affirm *A*. In this way, we can think of affirmation and denial as two opposite ways of standing with respect to a content, positively and negatively. A "content" here, should be understood just as something that can be affirmed or denied.[20] For

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[20]I am thus construing "content" here in such a way that it is completely compatible with taking contents to themselves be intrinsically forceful. Indeed, a content can be construed as an act of assertion and affirmation and denial can thus be construed as two ways of standing with respect to such an act. After all, if I make an assertion, you can say "Yes, that's true," or "No, that's false," with respect to it, agreeing with it or disagreeing with it, affirming it or denying it. This is how I myself would like to understand contents. I don't impart this way of thinking here, however, so as to not completely abandon Frege.

The minimal way in which I am construing content here, as just something that can be affirmed or denied, is not, I don't think, alien to Frege. I take it that Conant agrees. In his 1991 paper, "The Search for Logically Alien Thought," Conant writes, "[F]or Frege, to grasp a thought is to be faced with the demand for judgment. It is to be faced, that is, with the question of whether the thought is to be affirmed or denied," (136). He goes on to add in a footnote, "In order to make it clear that affirming and denying do not comprise two different kinds of judging, i.e., two distinct sorts of acts, Frege will prefer to say: in the demand for judgment, we are faced with the question whether the thought or its negation is to be recognized as a truth," (173). But Frege is not always so clear. For instance, in *Logic*, he says what it is for some thought to be negation of a thought by saying that, when one holds that thought to be false, one holds this opposite thought to be true. He writes, "When it is a question of whether some thought is true, we are poised between opposite thoughts, and the same act which recognizes one of them as true recognizes the other as false," (1897/1979, 149). Here, Frege speaks of not only of two opposite thoughts, a thought and its negation, but also two opposite acts, an act of recognizing a thought as true and an act of recognizing a thought as false. He says that, when we're poised between two opposite thoughts, the act of recognizing one thought as true and the act of recognizing the other thought as false are, in some sense, "the same." These acts cannot be logically equated in this way if they cannot be independently logically identified. Now, the act of recognizing a thought as true, for Frege, is, of course, the act of judgment. What is the act of recognizing a thought as false? That, I'm claiming, is the act of denial, which is the logical compliment to the act of judgment. In what follows, I draw from Irad Kimhi (2018) and think of the capacity for judgment as a two-way capacity, where the act of affirming some content (the act of judgment) is a positive actualization of that capacity, and the act of denying that content is the corresponding negative actualization of the same capacity. The thought Frege expresses here thus becomes the following: when it is a question of whether some thought is true, we are poised between that thought and its negation, and in the very same act in which one positively actualizes the capacity for judgment with respect to one, one negatively actualizes that capacity with respect to the other.

any content, you can stand in either of these two ways with respect to it, but you cannot stand both ways with respect to it. That would be incoherent. If we think of ourselves as coherently maintaining positive and negative stances with respect to contents that can be affirmed or denied, we can articulate the sense of the logical notation by articulating which sets of stances we are able or unable to take if we take some stance to a logically complex symbol of the *Begriffsschrift*. That, I take it, is what Frege actually does in the *Begriffsschrift* in explicating the sense of his logical notation in terms of affirmation and denial. It is the conception of logic, implicit in this way of giving sense to the logical notation, that I now want to make explicit.

The way of thinking about Frege's logical notation that I want to make explicit will be easiest to articulate if I make use of the first-person pronoun "I," so that's what I'll do, articulating the sense of the logical notation from "the first-person perspective" of someone whose capacity for judgment is in act in such a way that it does not disagree with itself.[21] From this perspective, consider, for two contents *A* and *B*, four possible cases:
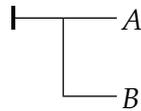
1. *A* is affirmed and *B* is affirmed.
2. *A* is affirmed and *B* is denied.
3. *A* is denied and *B* is affirmed.
4. *A* is denied and *B* is denied.

These four different "possibilities" consisting in some contents being affirmed

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[21]Though I articulate the sense of the logical notation that enables us to make the judgments we are thereby able to make from the first-person perspective, using the first-person pronoun "I," it is important to need to keep in mind, as van der Schaar says, "These judgements are thus made from a first-person perspective, but they are non-personal at the same time," (240). I should add, I don't think that it's actually necessary to the use the first-person singular here. I think the first-person plural, "we," might also be able to do the same work, or evern the third-person generic "one."

and some being denied are four different ways that I might non-neutrally stand with respect to the contents A and B. For these two contents, I might affirm the first and affirm the second, affirm the first and deny the second, deny the first and affirm the second, or deny the first and deny the second. If I have not yet affirmed or denied any contents involving the contents A and B, all of these ways of standing with respect to these contents remain possibilities for me. They are all ways that I am able to stand without being in disagreement with myself. As I affirm or deny some contents, I preclude myself from being able to stand some of these ways. The following expression:

$$\vdash\!\!\begin{array}{l} \rule{1em}{0pt}A \\ \rule{1em}{0pt}B \end{array}$$

expresses the an act of affirming a complex content, where this act of affirmation amounts to ruling out the third of the above possible stances, the one in which B is affirmed and A is denied. If I affirm the complex content,

$$\begin{array}{l} \rule{1em}{0pt}A \\ \rule{1em}{0pt}B \end{array}$$
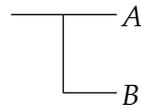
I rule out this set of stances as a possibility for myself, precluding myself from being able to both affirm B and deny A. Accordingly, if I take this stance and if I affirm B, then I cannot deny A. On the flip side, if I deny,

$$\vdash\!\!\!\!\begin{array}{l} \rule{1.2em}{0.4pt}\, A \\ \rule{1.2em}{0.4pt}\, B \end{array}$$
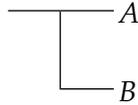
this *amounts to* affirming *B* and denying *A*. Accordingly denying this complex content precludes me from being able to either deny *B* or affirm *A*, for doing either such thing would be to disagree with myself.

Having introduced just this much notation, let us consider again what sense we can make of what it is for a formula of Frege's *Begriffsschrift* to express a logical truth. The core principle that enables us to comprehend a formula as expressing a logical truth is just this: I cannot disagree with myself. That is, I am unable to both affirm and deny a single content. If a content is such that there is no way that I can deny it, since to do so would be to disagree with myself, then, if I am to proceed in the activity of judgment with respect to it, I cannot do anything but affirm it. Such an act of affirmation and only such an act can be expressed as a formula of *Begriffsschrift*. To consider again the case we considered earlier, let ϕ and ψ be any two contents that can be affirmed or denied, and consider the following formula:

(1)

$$\vdash\!\!\!\!\begin{array}{l} \rule{1.2em}{0.4pt}\, \phi \\ \rule{1.2em}{0.4pt}\, \psi \\ \rule{1.2em}{0.4pt}\, \phi \end{array}$$
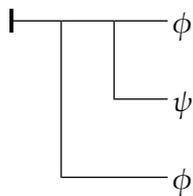
(1) rules out the possibility in which

$$\underline{\qquad}\ \phi$$

is affirmed and

$$
\begin{array}{l}
\underline{\qquad}\!\!\rule[0.5ex]{0.01in}{0.5in}\!\!\underline{\qquad}\ \phi \\
\qquad\quad\underline{\qquad}\ \psi
\end{array}
$$

is denied. Recall, if I deny the second content, this amounts to affirming ψ and denying ϕ. Clearly, if I affirm ψ and deny ϕ, then I deny ϕ. So, if I deny the second content, I deny ϕ. But if I affirm the first content, then I affirm ϕ. Thus, if I affirm the first content and deny the second content, I both affirm and deny ϕ. But I cannot both affirm and deny a single content; that is just what I am unable to do. Accordingly, I cannot affirm the first content and deny the second content. This is just the way of standing with respect to these contents that I rule out when I write down (1).

A formula in which a vertical stroke is prefixed to a complex symbol expresses the act of affirming, rather than denying, what is expressed by that complex symbol. Now, the complex symbol to which the vertical stroke is prefixed in (1) is the following:

$$
\begin{array}{l}
\qquad\quad\underline{\qquad}\ \phi \\
\qquad\quad\underline{\qquad}\ \psi \\
\underline{\qquad}\ \phi
\end{array}
$$

The content expressed by this symbol is such that, once I grasp what it would be to affirm or deny it, I recognize that I cannot deny it. To deny this content

29

would be to affirm

$$\mathstrut\rule{1.5em}{0.4pt}\,\phi$$

and deny

$$\phi$$
$$\psi$$

and, as we've already seen, to do this would be to both affirm ϕ and deny ϕ, and that is something that I cannot do. Since I am unable to deny this content, the only way that I can stand with respect to it, if I am to meet the demand for judgment, is to affirm it. If I am to actualize the capacity for judgment with respect to this content, I cannot do anything but perform act expressed by (1), and so that is what I do in writing down (1):

(1)

$$\phi$$
$$\psi$$
$$\phi$$

It should be clear, the description that I've given, that (1) itself does not express any particular individual's act of affirmation, either my own or anyone else's. Rather, it expresses a general act of affirming a content, a content one that admits of no coherent denial. To deny the content that is affirmed in the act expressed by (1) would be to take a set of stances with respect to the contents ϕ and ψ such that one disagrees with oneself. So, (1) expresses an act that admits of

30

no coherent opposite. To spell this out a bit, we might draw on the Aristotelian toolkit recently drawn on this connection by Irad Kimhi (2018), and think of the act of affirming some content and the act of denying that content as two opposite ways in which a single capacity, the capacity for judgment, can be actualized with respect to that content.[22] The capacity for judgment is what Aristotle calls a "two-way capacity" (*Met.* Θ-2) and it can be actualized positively, in the affirmation of some content, or negatively, in the denial of that content. In writing down (1), I express a positive actualization of the capacity to judge which is such that there is no coherent corresponding negative actualization.[23] On this way of thinking, when I, using Frege's logical notation, write down a formula contains the vertical stroke, I express a way for the capacity to judge to be positively in act such that the corresponding negative act is not a way for it to be coherently in act at all. Insofar as the activity of the capacity proceeds in such a way that it agrees with itself, it cannot be in act in any way that is in opposition to the mode of activity expressed by the formula I write down.

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[22]It is important to note that the way I am bringing in this Aristotelian notion to bear in the explication of the notation of classical propositional logic is *not* the way that Kimhi brings it to bear in this context. Whereas I am construing affirmation and denial as two opposite acts of a two-way capacity, Kimhi construes the two opposite propositions ϕ and $\neg\phi$ as two opposite acts of a two-way propositional capacity (2018, 60-61). The main difference is that I am maintaining a distinction here between force and content that Kimhi wants to reject. Now, Kimhi takes that, if one makes a distiction betwen acts and contents, there are two ways that one can account for the contradictoriness of the pair $\langle\phi,\neg\phi\rangle$. On the one hand, one can, like Descartes, take two contradictory judgments to have the same content but opposite force: one judgment is the affirmation of ϕ and one judgment is the denial of ϕ. On the other hand, one can, like Frege, take two contradictory judgments to have opposite contents but the same force: one judgment affirms ϕ and one judgment affirms $\neg\phi$ (2018, 60). Here, I am suggesting that we can have it both ways. By taking the capacity to judge to be a two-way capacity, we can take it that, in any act in which it is positively actualized with respect to ϕ it is, in one and the same act, negatively actualized with respect to $\neg\phi$. Accordingly, two contradictory judgments disagree both with respect to force *and* with respect to content: one act affirms ϕ and denies $\neg\phi$ and the other act denies ϕ and affirms $\neg\phi$. This is not a possibility Kimhi considers. Now, my own inclinations actually do side with those of Kimhi, but it seems to me that going in for this possibility is open for my Kantian reconstrunction of early Frege.
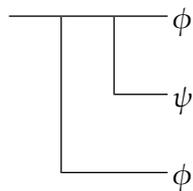
[23]According to Kimhi, fully thought through, there is no corresponding negative actualization at all. I will return to this thought in the next section.

To write down a formula that contains a vertical stroke is not to express some particular individual's act of affirming some content, but to express a generic act of affirming some content, a "generic instance of the exercise of a general power" (Conant forthcoming, 360), the power of judgment. However, to write down a formula that contains the verticle stroke is not just to *express* the act of affirming some content, but also to *affirm* that content oneself. So, though (1) *itself* does not *express* a particular individual's act of affirmation, my act of *writing down* (1) *constitutes* a particular individual's act of affirmation, in particular, my own. The act that is manifested by the use of the logical symbolism is thus, in Conant's words, "at one and the same time *general* and *actual*," (360), general insofar as a formula that contains the judgment stroke *expresses* a *generic* act of affirmation, actual insofar as my act of writing down such a formula *is* an *actual* act of affirmation. In using the logical notation, I am, at one and the same time, expressing general ways for the capacity to judge to be in act and actualizing my capacity to judge in these ways. The use of Frege's logical notation brings to explicit consciousness a conception of what it is for the two-way capacity to judge to be coherently in act in the judgment of logically complex contents, where "our very conception of the power depends not only upon our conceiving the actuality of its exercise as a condition of its reality, but upon our actually exercising it," (360).

4   Logical Necessity and Rational Compulsion

We considered above the way in which Frege's mature conception of the nature of logic is what Conant calls a "Leibnizian" rather than "Kantian" conception of logic, and, on such a conception, the judgment stroke is out of place. I've now

claimed that there were the seeds of a Kantian conception of logic in Frege's early explication of his logical notation, and I have tried to flesh that conception out a bit and show that, on this conception, the judgment stroke is not at all out of place. I want to close by bringing out the fundamental difference between these two conceptions of logic by keying in on the two different flavors of modal vocabulary that I used to explicate them. The flavor of modal vocabulary that I employed in explicating what it is for a formula of *Begriffsschrift* to be a logical truth, on Frege's mature Leibnizian conception, was that of *alethic* modality: I principally spoke in terms of what is "possible" or "impossible." On a Leibnizian conception of Frege's logic, a content is a logical necessity just in case it is not possible for that content to be false. It is not possible for

$$
\begin{array}{c}
\rule{3em}{0.4pt}\!\!\!\begin{array}{l} \rule{2em}{0.4pt}\,\phi \\[1ex] \rule{1em}{0.4pt}\,\psi \end{array} \\[2ex] \rule{2em}{0.4pt}\,\phi
\end{array}
$$

to be false in the sense that, for any contents at all that we plug in for ϕ and ψ, this thought is true. So this truth is necessary in the sense of being a truth about every pair of thoughts that there is or could possibly be. By contrast, the flavor of modal vocabulary that I employed in explicating what it is for a formula of *Begriffsschrift* to be a logical necessity, on Frege's incohate Kantian conception, was that of *agentive* modality (Mandelkern et. all, 2017): I principally spoke in terms of what I am "able" or "unable" to do. On a Kantian conception, an act of judgment is a logical necessity in case I am unable to actualize my capacity for judgment in a way that is opposed to that act. I am unable to deny

33

$$\vdash \!\!\!\!\!\! \begin{array}{l} \phi \\ \psi \\ \phi \end{array}$$
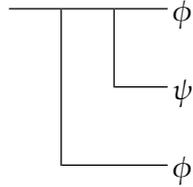
in the sense that denying it would amount to both affirming ϕ and denying ϕ, and that is something that I am unable to do. Since I am unable to deny this content, if I am to proceed in actualizing my capacity for judgment with respect to it, I cannot but affirm it, and that is what I do by writing down the following formula:

$$\vdash \!\!\!\!\!\! \begin{array}{l} \phi \\ \psi \\ \phi \end{array}$$
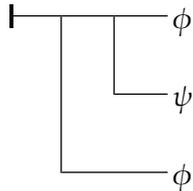
On the Leibnizian conception, any sense in which the act of judgment expressed by this formula can be said to be "necessary" is not the sense in which the content judged can be said to be "necessary." The act of judgment is *normatively* necessary and the content judged is *alethically* necessary. On the Kantian conception, by contrast, there is one sense of necessity that applies to both the content and the act: *agentive* necessity, where our understanding of the necessity of a content goes through our understanding of the necessity of the act of affirming it, and our understanding of the necessity of the act of affirming it goes through our actually performing this act with the use of the judgment stroke.

But what is this notion of "agentive necessity" of which I speak here? It is, I take it, just the dual of agentive possibility, or ability. For some possibility operator $\diamond$, its dual necessity operator, $\square$, can be defined as $\neg\diamond\neg$, where $\neg$ is a formal negation operator. For instance, for something to be alethically necessary is for it not to be possible for it not to be the case (the term "necessity" is often just reserved for this flavor of necessity). For something to be normatively necessary (that is, "obligatory") is for one to not be permitted not to do it. For something to be agentively necessary, then, is for one not to be able not to do it. It is this status agentive necessity, the dual of ability, that we express when we say that we "cannot but" do something.[24]

In their paper "Agentive Modals," Matt Mandelkern, Ginger Schulteis, and David Boylan (2017) identify this dual of the modal status of ability as the status of *compulsion*. To be *compelled* to do something, they say, is to not be able not to do it. There is reason, however, to be at least a bit trepidatious about the use of the term "compulsion" in the present context.[25]

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[24]This formulation may make it seem that the logical form is actually $\neg\neg\diamond$ rather than $\neg\diamond\neg$, but this is because the surface grammar of "cannot" is misleading. To say that someone "cannot φ" is not to say that they are able not to φ (as the phrase "can not φ" would suggest), but, rather, to say that they are not able to φ. So, to say of someone that they "cannot not φ" is to say that they are not able not to φ (i.e. they are compelled to φ).

[25]It's worth also noting, however, that the notion of compulsion is not at all foreign to this context. While the standard way to characterize the entailment relation that obtains between a set of premises and a conclusion that deductively follows fro them is in terms of the statuses of alethic modality, saying that, if all of the premises are true, it is not possible for the conclusion to be false, one can also characterize the entailment relation that obtains between a set of premises and a conclusion that is deductively entailed by them *agentively*, by saying that someone who accepts the premises is, in a certain sense, *compelled* to accept the conclusion. For a clear expression of this of the sort of compulsion, consider Lewis Carroll's (1895) classic article "What Achilles Said to the Tortoise." Towards the end of the article, the Tortoise asks Achilles what happens if he accepts A and B and C and D, a set of premises that deductively entail a conclusion Z, but "refuses" to accept Z. Achilles replies to the Tortoise as follows:

> Then Logic would force you to do it! [...] Logic would tell you 'You can't help yourself. Now that you've accepted A and B and C and D, you must accept Z!' So you've no choice, you see, (280).

Achilles' "must" here is the "must" of agentive modality. It is a compulsion modal. When

In his 1991 article, "The Search for Logically Alien Thought," Conant uses the language of compulsion only once, and it is to characterize the incoherent set of commitments taken on by the psychologistic logician. Conant writes,

> The incoherence lies in the psychological logician's saying at one moment, "We cannot but take these laws to be true," and in the next, disparaging them as only true for us—if we are *compelled* to take them as true, then we take them to be true; and hence we *must* [...] regard anyone who denies them as in the wrong. The psychologistic logician, Frege says, "presumes to acknowledge and doubt a law in the same breath," (146).

I take it that Conant uses the vocabulary of compulsion only in characterizing the views of the pysychologistic logigian and not in characterizing his own views because he takes it that this vocabulary suggests a certain conception of the necessity of logical laws that he wants to reject, one in which we are forced to judge them by something imposed on us from the outside, be it our psychological constitution or anything else. If I say, for instance, that I "cannot but" affirm
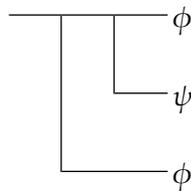
—that I am "compelled" to affirm it—I run the risk of making it sound as if I could try to deny it, but the there is something—my psychological constitution,

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Achilles says to the Tortise "you must accept Z," he aims to be expressing the fact that the Tortoise is *compelled* to accept Z, and he makes this clear by glossing what he says by saying "you can't help yourself." Now perhaps, in the case of the Achilles and the Tortoise, talk of compulsion is not out of place. It does seem that we can speak of ourselves, if we've affirmed some set of things, A, B, C, and D, and clearly entail some other thing, Z, as being compelled to affirm Z, insofar as we continue to affirm A, B, C, and D.

perhaps—preventing me from being able to deny it and forcing me to do the opposite, affirm it. That would be psychologism, and, if there is any view that Conant has maintained from 1991 to now, it is that this is not a position we can coherently hold. If the only way to hear the word "compulsion," is one such that, if we used the term in speaking of our relation to logical laws, we would commit ourselves to psychologism, then we should not use the word in this context. But perhaps there is another way of hearing this term in this context.

Set aside the term "compulsion" for a moment and consider just the phrase "cannot but." When I say, for instance, "If I eat one pringle, I cannot but eat another," I am suggesting that there is something I could try to do—refrain from eating another Pringle—but something (an overpowering desire, perhaps) is preventing me from doing this, forcing me to eat another Pringle. This is how Mandelkern et. all principally hear the phrase.[26] But this is not how we should hear the phrase "cannot but" in the context of logic. When I say "If I am to procede in judgment with respect to it, I cannot but affirm the content shown above," I do not mean to be suggesting that there is something preventing me from denying it, forcing me to affirm it. Denying this content is something that I am unable to do, not in the sense I could try to do it but there is something stopping me from being able to do so, but in the sense that, once I'm clear on things, I see that there's no "it" to be done.[27] To deny this content would be to affirm ϕ and deny ϕ in a single act of judgment, and affirming and denying a

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[26]Their analysis of the meaning of the phrase makes this way of hearing it explicit. On their account, the act-conditional analysis of agentive modals, a sentence of the form "S is able to φ" is true if there is some practically available action (something within S's power) such that, if S does it, S φs (314). The dual, then, "S is not able not to φ" will be true just in case there is no practically available action such that, if S does it, S does not φ. On this account, a compulsion modal expresses that preventing oneself from doing something is not within one's power.

[27]On this point, consider a quote from Wittgenstein that Conant (both 1991 and current) takes as a guiding principle in thinking about logical necessity: "The great difficulty here is not to represent the matter is if there was something one could not do," (Wittgenstein 1953, §374, 123e).

37

single content in a single act is not a way for the capacity for judgment to be put to work.[28] It can be put to work positively, in affirming ϕ, or it can be put to work negatively, in denying ϕ, but it can't be put to work in both ways at once. That's just part of what it is for it to be a two-way capacity. Accordingly, I can't actualize the capacity for judgment in a way that would constitute an act of denying this content because there is simply no way for the capacity for judgment to be in act that would constitute my doing so. Since there is nothing that it could be to deny this content, if I am to proceed in judgment with respect to it, I cannot do anything but affirm it. Now, if we just let the term "compulsion" expresses the dual of ability (after all, we need some word to express this dual), then it would be right to say that, when faced with the demand for judgment, I am "compelled" to affirm this content. We must be clear, however, that this is a use of "compulsion" in which there is no implication of there being any sort of force from the outside. In some cases of compulsion there is, but, in some cases of compulsion, there is not. Compulsion without any force from the outside, I take it, is just what rational compulsion is.

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