Supposition: No Problem for Bilateralism

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Abstract

In a recent paper, Nils Kürbis argues bilateral natural deduction systems in which assertions and denials figure as hypothetical assumptions are unintelligible. In this paper, I respond to this claim on two counts. First, I argue that, if we think of bilateralism as a tool for articulating discursive norms, then supposition of assertions and denials in the context of bilateral natural deduction systems is perfectly intelligible. Second, I show that, by transposing such systems into sequent notation, one can make perfect sense of them without talking about supposition at all, just talking in terms of relations of committive consequence. I conclude by providing some motivation for adopting this normative interpretation of bilateralism on which this response to Kürbis's argument is based.

keywords: bilateralism, assertion, denial, supposition, assumption, speech acts

0 Introduction

A bilateral system of logic provides rules for manipulating positively or negatively signed formulas. The standard way of thinking about the formulas that figure in bilateral systems, explicated by such authors as Smiley [25] and Rumfitt [19], is to think of the signs as expressing two opposite speech acts: assertion and denial.[1] Thus, a formula of the form $+A$ is taken to express the *assertion* of A whereas $-A$ is taken to express the *denial* of A. This approach has been prominent in recent developments of inferentialist semantics. However, it has recently come under fire by Nils Kürbis [16].[2] In his recent paper "Supposition: A Problem for Bilateralism," Kürbis claims that the notion of supposing an assertion or a denial makes no sense, as it involves embedding one speech act (assertion or denial) under another (supposition). Just as asserting a denial makes no sense, Kürbis claims that supposing an assertion makes no sense either. Since bilateral natural

[1] Various other terms for these speech acts have been deployed, such as "affirming" rather than "asserting," and "rejecting" rather than "denying." Little hangs on such differences for our purposes here.

[2] For other expressions of this same basic argument, see also [11, 230-231], [6, fn. 23], [14, 221], [18, 11-17], and [28, 4-5]. I focus on Kürbis's recent paper here since it is the most sustained development of this argument.

deduction systems of the sort proposed by Smiley and Rumfitt essentially feature such suppositions of assertions and denials, these systems, Kürbis claims, are unintelligible. In this paper, I will argue that, given the way bilateralism is actually understood by proponents of it in the context of inferentialist semantics (the main project in which it has actually be put to use), Kürbis's argument fails, and it does so on two counts. First, suppositions of assertions and denials of the sort that figure in these natural deduction systems can be interpreted in a way that makes perfect sense. Second, that, by transposing these systems into sequent notation, one can make perfect sense of these systems in a way that does not appeal to supposition at all. So, supposition is no problem for bilateralism.

1 A Perfectly Intelligible Reading of Supposition

For our purposes, it will suffice to just consider the fragment of Rumfitt's bilateral natural deduction system consisting in the following operational rules:

$$\frac{-A}{+\neg A}\,+_{\neg I} \qquad \frac{+\neg A}{-A}\,+_{\neg E} \qquad \frac{+A}{-\neg A}\,-_{\neg I} \qquad \frac{-\neg A}{+A}\,-_{\neg E}$$

$$\frac{+A \quad +B}{+A \wedge B}\,+_{\wedge I} \qquad \frac{+A \wedge B}{+A}\,+_{\wedge EL} \qquad \frac{+A \wedge B}{+B}\,+_{\wedge ER}$$

and the following pair of bilateral structural rules:[3]

$$\frac{+A \quad -A}{\bot}\ \text{Incoherence} \qquad \begin{array}{c}\overline{+A}^{\,u}\\ \vdots \\ \dfrac{\bot}{-A}\ \text{Reductio}_{+}^{\,u}\end{array} \qquad \begin{array}{c}\overline{-A}^{\,u}\\ \vdots \\ \dfrac{\bot}{+A}\ \text{Reductio}_{-}^{\,u}\end{array}$$

The Incoherence rule says that from the assertion of A and the denial of A one can conclude an incoherence. The first Reductio rule says if, given the assumption of an assertion of A, one can conclude an incoherence, then one can discharge that assumption and conclude the denial of A, whereas the second Reductio rule says that if, given the assumption of a denial of A, one can conclude an incoherence, then one can discharge that assumption and conclude the assertion of A. This fragment of Rumfitt's system constitutes a sound and complete proof system for classical logic in that an argument with premises $A_1, A_2 \ldots A_n$ and conclusion B is classically valid just in case this system proves $+B$ from $+A_1, +A_2 \ldots + A_n$.[4] To

[3] This way of splitting up structural rules, which are combined in the presentations of Smiley and Rumfitt, follows the presentation of Incurvati and Schlöder [10]. They call the principle I call "Incoherence" "Rejection."

[4] See [10, 754]. Though they establish this result for a somewhat different system in which $-$ expresses *weak* rejection, the same result holds in the same way for this system.

see how this sort of system works, let us look at a simple proof which involves the assumption of a signed formula. Consider, for instance, the proof of $+\neg(p \wedge q)$ from $-q$,

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{+p \wedge q}^{\;1}}{+q}\;{+}_{\wedge_{ER}} \qquad -q}{\bot}\;\text{Incoherence}}{-p \wedge q}\;\text{Reductio}_{+}^{\;1}}{+\neg(p \wedge q)}\;{+}_{\neg_I}$$

In this proof, we assume $+p \wedge q$, derive an incoherence, and so discharge our assumption and write down $-p \wedge q$, from which we are able to conclude $+\neg(p \wedge q)$.

Kürbis claims that there is no intelligible reading of the above proof according to which "+" expresses assertion and "−" expresses denial. Such a reading, he claims, would involve thinking of one speech act—assertion—as embedded within another—supposition. But this, Kürbis claims, is unintelligible; just as denying an assertion makes no sense, supposing an assertion doesn't make sense either. So, bilateral proof systems of the sort proposed by Smiley and Rumfitt are unintelligible. As a consolation to bilateral logicians who don't *take themselves* to be writing nonsense in using bilateral proof systems, he offers the following error theory:

> My best diagnosis is that the practice of bilateral logicians shows that their + and − are nonembeddable truth and negation operators. The description of − and + as speech acts does not match their use. [16, 23]

As a bilateral logician, I can report firsthand that this is not how I am using + and −. To be clear, I acknowledge that it is *possible* to read singed bilateral systems in this sort of alethic way, with the "two ways" of bilateralism being interpreted as truth and falsity, the two signs expressing these two opposite truth values.[5] Indeed, this is how the signs are used, for instance, in Smullyan's [26] signed tableaux system.[6] I will return to this interpretation in the final section for the paper. For the moment, however, I will just report that I don't use the signs of

[5]For an illuminating account of the relation between normative bilateralism (of the Restall/Ripley sort) and truth-maker semantics, see Hlobil [7]. Though Hlobil is considering different formal systems, the general normative/alethic correspondence (a philosophical account of which is developed at length by Brandom [2]) applies here as well.

[6]Fun fact: if you take the fragment of Rumfitt's bilateral natural deduction system consisting in solely the elimination rules, and you tweak the positive conditional rules so that they are of the same form as the positive disjunction (or negative conjunction) rules, and you do every proof by Reductio, then this system just is a notational variant of Smullyan's signed tableaux system.

bilateral logic to express truth and falsity; I use them to express assertion and denial.

Before saying, exactly, where I take the error in Kürbis's argument to lie, let me first just say how I read proofs in bilateral natural deduction systems. Here is how I propose we read the above proof, following Incurvati and Schlöder [10] [9] in thinking of the horizontal line of the natural deduction system as expressing a relation of committive consequence:

> Suppose we assert $p \wedge q$. Then we're committed to asserting q. But we deny q. Incoherence. So, given that asserting $p \wedge q$ leads to an incoherence, we're committed to denying $p \wedge q$, and thus, to asserting $\neg(p \wedge q)$.

This reading seems perfectly intelligible to me. According to this reading, when we write down $+p \wedge q$ as an assumption in the context of the above proof, this is not to be read as not "Suppose *Yes, $p \wedge q$!*" (or something to that effect), but, rather "Suppose we assert that $p \wedge q$" (or something to *that* effect). We then reason about what we're committed to asserting or denying, given that hypothetical supposition. If we conclude that we're committed to asserting and denying the very same thing, given that assumption, we can discharge that assumption and conclude that we're committed to the opposite speech act. Of course, the use of first-personal "we" in articulating the significance of such a proof is optional. We might just as well read it in third-personal generic terms, conceiving of it as telling us that anyone who denies q is committed to asserting $\neg(p \wedge q)$. Reading it in this way, we can read it as follows:

> Consider someone who denies q. Now suppose that they assert $p \wedge q$. Then they're committed to asserting q. But they deny q. Incoherence. So, given that asserting $p \wedge q$ leads to an incoherence, they're committed to denying $p \wedge q$, and thus, to asserting $\neg(p \wedge q)$. Thus, someone who denies q is committed to asserting $\neg(p \wedge q)$.

Whatever the specific vocabulary one prefers to deploy to spell out a reading of this sort, Kürbis claims that this sort of reading, according to which a supposition of a positively signed formula is read along the lines of "Suppose it is asserted that A" is unavailable to the bilateralist. He makes two points that are supposed to establish this. Neither of them do.

The first point Kürbis makes is that supposing that we assert A is distinct from supposing A (16). That is, of course, true. After all, one is not asking someone to suppose something contradictory when one says "Suppose that we assert A and suppose further that it's not the case that A," but one is asking someone to suppose something contradictory when one says "Suppose that A

and suppose further that it's not the case that A." So there is indeed a crucial distinction between supposing an assertion of A and supposing A itself. And it's true, on this reading of speech act bilateralism, what one is supposing in the context of a hypothetical proof is the first sort of thing, not the second sort of thing. As the above explication of this proof makes clear, when one writes down $+A$ in the context of a hypothetical proof as we do above, one is *not* supposing that A. Rather, we are supposing that we *assert* that A, and we then reason about what we're committed to asserting or denying given that supposition. So I acknowledge Kürbis's first point, but acknowledging this point does not itself raise any problem for this reading. On the contrary, this seems to be just what one should say on a normative understanding of bilateralism, according to which bilateral logic concerns the norms governing assertions and denials in a discursive practice.

The second point that Kürbis makes is that, when the bilateralist writes down $+A$ in the context of a proof, such formulas are "not reports that any such speech acts have been performed or assertions that they could be performed," (17). Of course, that is also true, but, once again, there is no reason that the proponent of the proposed reading must disagree with this claim. The activity one is engaged in when one uses a bilateral natural deduction system is not an activity of reporting what any particular individuals have asserted or denied, nor is it an activity of reporting which speech acts are possible (at least, in the alethic rather than deontic sense of "possible"). Rather, it is a way of articulating what speech acts anyone at all who traffics in assertion and denial of logically complex sentences is *committed* to, either hypothetically, given other assertions or denials, or categorically. This is a normative enterprise, not a descriptive one. When one says, for instance, "If one denies q, then one is committed to affirming $\neg(p \wedge q)$" this is to be understood by analogy to my saying "If one moves one's king, then one can't go on to castle." In saying that latter thing, I am not reporting anything about any particular chess players. I am, rather, expressing the rules of the game of chess, in particular, how the act of moving one's king is normatively related to the act of castling. Likewise, if I say "If one denies q, then one is committed to asserting $\neg(p \wedge q)$," I am not reporting anything about any particular speakers. I am, rather, expressing the rules of the "game of assertion and denial" in a language that contains negation and conjunction, in particular, how the act of denying q is normatively related to the act of asserting $\neg(p \wedge q)$.

Concretely, then, my diagnosis of Kürbis's argument is that it rests on the following false dichotomy: either a formula of the form "$+A$," as it is used in the context of a bilateral natural deduction system, functions as a way for the logician to actually perform the speech act of asserting A themself or it functions to predicate the act of asserting of A of some particular speaker. Given that it is clearly not doing the latter, Kürbis assumes it must be doing the former, and given that the logician clearly *is* performing the speech act of *supposing* when they write down "$\overline{+A}^{\,1}$" in the context of a natural deduction system, Kürbis concludes that the use of bilateral natural deduction involves an incoherent embedding of speech acts. However, there is a third possibility for understanding the function of a formula such as $+A$ in the context of a bilateral natural deduction system: such a formula *simply expresses the act of asserting A*, not in the sense that involves the actual performance of that act on the part of the logician, but in just the sense that the sentence "A" expresses the proposition that A or the predicate "round" expresses the property of being round.[7] In English, such an act might be expressed with the gerund "Asserting A" (as in "Asserting A commits one to denying $\neg A$"), the definite description "The assertion of A" (as in "The assertion of A commits one to the denial of $\neg A$"), or the declarative sentence "One asserts that A" (as in "If one asserts that A, one is committed to denying $\neg A$"). To suppose such a speech act, in the context of a natural deduction system, is to suppose its performance, and thus, the third sort of expression is most naturally used when intuitively explicating the sense of the bilateral notation (saying, "Suppose one asserts that A . . . "). Crucially, however, this is not to suppose that it is actually performed by anyone in particular. Rather, it is simply to suppose it is performed, enabling one to reason hypothetically about the consequences of such a performance so as to arrive at an understanding of the various relations of the relations of committive consequence that obtain between the various acts of assertion and denial that might be performed in a discursive practice.

2 Doing without Supposition

Let me now spell out this normative conception in a bit more detail, which will enable us to transpose the above thoughts into an equivalent notation in which no talk of supposition is needed. Following Brandom [1], I think of the speech acts of assertion and denial as "moves" that one might make in "the game of giving and asking for reasons," (xviii). Whereas assertion is the basic sort of move that one might make, functioning to entitle others to make that assertion, denial is the basic sort of counter-move, functioning to challenge an assertion. On the proposed reading of bilateral natural deduction, the deducibility relation is understood as a relation of *committive consequence*, where, once again, this notion is understood along the lines proposed by Brandom [1, 157-166]. Thus, I read a sequent of the form $\Gamma \vdash \varphi$ (where Γ is a set of signed formulas and φ is a single signed formula) as saying that *making* the moves in Γ, be they assertions or denials, *commits one* to φ, be it an assertion or denial. For instance,

[7] For discussion of the Fregean sense of "express," see for instance, [5, 17-18]. I suspect that it is this ambiguity in different uses of "express" that has led to some of the confusion underlying

$-q \vdash +\neg(p \wedge q)$ says that denying q commits one to asserting $\neg(p \wedge q)$. Likewise, $\vdash +\neg(p \wedge \neg p)$ says that one is simply committed to asserting $\neg(p \wedge \neg p)$. In the context of Weakening, this latter sequent can be understood as saying that one is committed to asserting $\neg(p \wedge \neg p)$ no matter what else one asserts or denies. Thus, on this reading, everyone, regardless of what moves they've made, is committed to the assertion of every tautology, and everyone is committed to the denial of every contradiction. It is important to be clear, however, that one's being *committed* to an assertion does not mean that one must actually *make* that assertion. As Restall [17] says "that way lies madness, or at least, making too many assertions," (82). The notion of being committed to asserting some sentence is indeed a kind of obligation. Crucially, however, it's a sort of *dispositional* obligation, one which can *triggered* in various circumstances, rather than a *standing* obligation. Specifically, if one is committed to an assertion, then one is obligated to actually make that assertion *if one is appropriately prompted to do so* in a dialogical context.

This conception of committive consequence allows us to interpret sequents as directly expressing such normative relations. Transposing a natural deduction system into sequent notation, we can think of the rules as telling us which relations of committive consequence obtain between various assertions and denials of logically complex sentences, starting from the basic principle that, regardless of whatever other moves one has made, making some assertion or denial commits one to that very assertion or denial. That is, we have the following axiom, where φ is any (positively or negatively) signed formula:

$$\frac{}{\Gamma, \varphi \vdash \varphi}\ \text{Reflex.}$$

The operational rules tell us, for instance, that if some set of assertions and denials Γ commits one to denying A, then Γ commits one to asserting $\neg A$, and so on. The coordination principles of a bilateral system can be understood as relating relations of committive *consequence* between assertions and denials to the *incoherence* of sets of assertions and denial. Using a sequent with an empty right-hand side to express the incoherence of the set of moves on the left, the co-ordination principles can be put as follows:

$$\frac{\Gamma \vdash +A \quad \Delta \vdash -A}{\Gamma, \Delta \vdash}\ \text{Inc} \qquad \frac{\Gamma, +A \vdash}{\Gamma \vdash -A}\ \text{Red}_{+} \qquad \frac{\Gamma, -A \vdash}{\Gamma \vdash +A}\ \text{Red}_{-}$$

Transposed into a sequent setting, Incoherence say that if making all of the moves in Γ commits one to asserting A and making all of the moves in Δ commits one to to denying A, then making all of the moves in $\Gamma \cup \Delta$ is incoherent. Reductio$_{+}$ says that if making all of the moves in Γ along with asserting A is incoherent, then making all of the moves in Γ commits one to denying A, whereas Reductio$_{-}$

says that if making all of the moves in Γ along with denying A is incoherent, then making all of the moves in Γ commits one to asserting A.

Having explicated sequent notation in this way, consider again the proof of $+\neg(p \wedge q)$ from $-q$ above, but now transposed into sequent notation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{+p \wedge q \vdash +p \wedge q}\,\text{Reflex.}}{+p \wedge q \vdash +q}\,+_{\wedge_{EL}} \qquad \dfrac{}{-q \vdash -q}\,\text{Reflex.}}{+p \wedge q, -q \vdash}\,\text{Inc}}{-q \vdash -p \wedge q}\,\text{Red}_{+}}{-q \vdash +\neg(p \wedge q)}\,+_{\neg_I}$$

We read this proof as follows:

> Asserting $p \wedge q$ commits one to asserting $p \wedge q$, and so asserting $p \wedge q$ commits one to asserting q. Denying q commits one to denying q. So, asserting $p \wedge q$ along with denying q is incoherent. Accordingly, denying q commits one to denying $p \wedge q$. Thus, denying q commits one to asserting $\neg p \wedge q$.

Given Weakening, we can take such a conclusion sequent as telling us that, no matter what other moves one has made, denying q commits one to asserting $\neg(p \wedge q)$. In this way, we can interpret the above proof without talking about supposition at all.

Now, of course, the fact that we don't need to *talk* of supposition in explicating a sequent-style natural deduction system of this sort does not itself mean that suppositional reasoning is not nevertheless still *there*, underlying our understanding of the system.[8] One might be tempted to think that a sequent of the form $\Gamma \vdash \varphi$, interpreted as expressing a principle of committive consequence in the way that I've suggested, is just a way of conditionally expressing that, under the supposition that some speaker makes the moves in Γ, we score them as committed to φ. I want to urge, however, that we should not understand talk of principles of committive consequence simply as covert talk of supposition. This is precisely because our possessing the scorekeeping principles that we do *grounds* our attribution of commitments in suppositional contexts. Consider, for instance, the committive consequence relation that obtains between asserting $p \wedge q$ and asserting p. Of course, given that we have this principle of scoring anyone who asserts $p \wedge q$ to be committed to asserting p, if we suppose that someone asserts $p \wedge q$, then, under this supposition, we'll score them as committed to asserting p. But the reason we attribute this commitment in this suppositional context is *because we have this principle of committive consequence*, and we

[8] I'd like to thank two anonymous referees for pressing me on this sort of challenge.

are applying it in this particular case. I suggest, then, that principles of committive consequence, possessed by discursive practitioners and in virtue which they keep discursive score as they do (in suppositional and non-suppositional contexts), might plausibly be regarded as *primitive* in explicating the conceptual significance of a logical system, and articulating a natural deduction system directly in terms of such relations of committive consequence enables one to explicate its conceptual significance without any appeal to supposition.[9]

Though I've focused my attention on bilateral natural deduction systems here, since these are the target of Kürbis's argument, in this context of thinking about a bilateral system as fundamentally concerned with relations of committive consequence, it is perhaps more natural to use a sequent calculus proper, with only introduction rules, rather than a natural deduction system with both introduction and elimination rules. Elsewhere [22], I've proposed the following sequent calculus for bilateral classical logic (where, φ is a signed formula and φ^* denotes the oppositely signed formula):[10]

$$\frac{}{\Gamma, \varphi \vdash \varphi}\ \text{Reflex.} \qquad \frac{\Gamma \vdash \varphi}{\Gamma, \varphi^* \vdash}\ \text{In} \qquad \frac{\Gamma, \varphi \vdash}{\Gamma \vdash \varphi^*}\ \text{Out (Red.)}$$

$$\frac{\Gamma \vdash -A}{\Gamma \vdash +\neg A}\ +_{\neg} \qquad \frac{\Gamma \vdash +A}{\Gamma \vdash -\neg A}\ -_{\neg}$$

$$\frac{\Gamma \vdash +A \quad \Gamma \vdash +B}{\Gamma \vdash +A \wedge B}\ +_{\wedge} \qquad \frac{\Gamma, +A, +B \vdash}{\Gamma \vdash -A \wedge B}\ -_{\wedge}$$

Rather than containing the Incoherence rule shown above, this system contains the converse of Reductio (called "In"), which is natural in a sequent setting, saying that, if Γ commits one to some move φ, then Γ along with the opposite of φ is incoherent.[11] In this system, the proof of $-q \vdash +\neg(p \wedge q)$ runs as follows:

[9] At the risk of stating the obvious, it's perhaps worth noting explicitly that this interpretation of natural deduction differs markedly from the interpretation of its original progenitors, Jaskowski [12] and Gentzen [4]. As Kürbis notes, both regard "making and discharging assumptions is essential to the process of logical inference as captured by natural deduction," [16, 11]. But the fact that the original progenitors of a logical system interpret the significance of a system in a certain way does not mean that future users of the system are forever bound to that interpretation. To appeal to a more recent authority in support of the thought that the system can be interpreted in the way that I've proposed here, note that the basic formal model for inferentialism appealed to by Brandom from the outset is Gentzen's natural deduction (see [1, 117-118]), but *nowhere* in Brandom's career-long development of inferentialism from [1] to [8] does he ever appeal to supposition in order to explain inferential norms; he simply speaks in terms of relations of committive consequence and normative incoherence as I have done here.

[10] There are, of course, a number of possible bilateral sequent calculi for classical logic. This is just one simple such system that makes use of coordination principles to contain only right introduction rules.

[11] The proof that the Incoherence rule is admissible in this system is essentially just a notational

$$\cfrac{\cfrac{\cfrac{\overline{-q, +p \vdash -q}\ \text{Reflex.}}{-q, +p, +q \vdash}\ \text{In}}{-q \vdash -p \wedge q}\ -_{\wedge}}{-q \vdash +\neg(p \wedge q)}\ +_{\neg}$$

We read this proof as follows:

> Denying q and asserting p commits one to denying q. So, denying q, asserting p, and asserting q is incoherent. Thus, denying q commits one to denying $p \wedge q$. So, denying q commits one to asserting $\neg p \wedge q$

Of course, when we recast the negative conjunction rule here in the "simplistic" logical notation and treat it as an introduction rule for a natural deduction systems (as I do in [24]), its use will involve making and discharging assumptions.[12] However, in this sequent context, there is no reason to treat this rule as involving making and discharging assumptions at all.

The above point can be made most persuasively, I think, if we consider the fact that it's an essential point of inferentialism that logical vocabulary can be deployed not just in the context of purely *logical* inferences (and incoherences), but also *material* inferences (and incoherences).[13] Consider, for instance, that asserting "It's red," asserting "It's ripe," and asserting "It's a blackberry" is (materially) incoherent. The negative conjunction rule tells us that, given this fact, asserting "It's red" commits one to denying "It's ripe and its a blackberry." There is no need to treat the incoherence of the three assertions, which can serve as the top sequent for an application of the negative conjunction rule, as involving any supposition at all; the three assertions are simply incoherent. Similar remarks can be made about the standard (positive) introduction rule for the (material) conditional. Given that asserting "It's red" along with asserting "It's ripe" commits one to denying "It's a blackberry" (and thus to asserting "It's not a blackberry), the conditional rule enables us to conclude that asserting "It's red" commits one to asserting "If it's ripe, then it's not a blackberry." The conditional functions to express the relation of committive consequence, and

variant on the Cut Elimination proof for Ketonen's [13] classical sequent calculus, with which this sequent calculus is equivalent, given the coordination principles of In and Out.

[12] In that context, the rule is displayed as follows:

$$\cfrac{\cfrac{\begin{matrix}\overline{+\langle A\rangle}\ u & \overline{+\langle B\rangle}\ v \\ \vdots & \vdots\end{matrix}}{\bot}}{-\langle A \wedge B\rangle}\ -_{\wedge_I}{}^{u,v}$$

[13] The centrality of material inferences in the context of inferentialism is most notably defended by Sellars [20].

this notion of committive consequence (to insist for the final time) need not be understood in terms of supposition. So, not only is the appeal to supposition in the context of natural deduction perfectly *intelligible*, as I've argued above, it is also *eliminable* in the context of a sequent system of the sort I've just laid out.

3 Why Go Normative at All?

I have articulated a reading of bilateral notation according to the bilateral logician uses this notation to explicitly articulate the normative relations that obtain between acts of assertion and denial. In this way, the notation is used by the logician to *talk about* acts of assertion and denial which may be performed by discursive participants, rather than to actually *perform* acts of assertion and denial themself. Indeed, I've suggested that *if* the bilateral logician is taken to be doing the latter thing, then Kürbis's argument goes through. But this is not what bilateral logicians such as myself or, for instance, Incurvati and Schlöder, are doing when we deploy a bilateral system.[14] Our development of bilateral proof systems is with the aim of articulating a normative pragmatic theory of content, articulating the *meanings* of linguistic expressions (such as "not" and "and") in terms of the *norms* governing their *use* in a discursive practice. Accordingly, it makes perfect sense, in the context of this project, to explicitly talk about speech acts of assertion and denial and the normative relations that obtain between them. Of course, a bilateral logician *can* use a bilateral natural system to determine the assertions and denials to which they are themself committed (given the various assertions and denials that they've actually made or to which they are committed), and, upon determining these commitments, come to explicitly make these assertions and denials. However, it would be utterly bizarre, for instance, in the context of a logic paper, for a bilateral logician to actually assert a sentence A themself by writing $+A$. That's simply not how the notation is used; once again, it's used for the bilateral logician to explicitly *reason about* normative relations between assertions and denials, not to *make* assertions and denials.[15]

[14] I mention Incurvati and Schlöder, since they explicitly endorse the sort of normative pragmatic inferentialism I sketch here (see especially [9, 35-62]). However, I take it that other prominent bilateralists, such as Francez, who less explicitly align themselves with this specific inferentialist program, can likewise be taken to have this general philosophical orientation.

[15] It's worth noting, in this regard, that my proposed interpretation of speech act bilateralism distances the bilateral logician's use of "+" from Frege's use of the vertical "judgment stroke," an analogy suggested by Rumfitt [19]. Frege's judgment stroke, as it's used in the *Begriffsschrift* for instance, plausibly *does* function as a means for the author to explicitly assert a formula. That is why Frege only prefixes the judgment stroke to logical truths. When he wants the reader to simply consider a formula (which may not be a logical truth), he uses the content stroke. Displaying contents in this way enables him to reason in natural language about the truth-possibilities of

I take myself to have done enough to defend speech act bilateralism against Kürbis's charge. The question might seem to remain, however, why one should adopt this approach to bilateralism, in which we are explicitly concerned with normative relations between speech acts, at all? As I mentioned above, it is completely possible to interpret bilateral logic in such a way that + and − express truth and falsity, and to think of the basic notions of consequence and incoherence in *alethic* rather than *normative* terms. Thus, using alethic notions, we might interpret the original proof shown in this paper as follows:

> Take it as given that it's false that q. Now, suppose it's true that $p \wedge q$. Then it must be true that q. But it's false that q. Contradiction. So, given that the truth of $p \wedge q$ leads to a contradiction, it must be false that $p \wedge q$, and thus, true that $\neg(p \wedge q)$.

This sort of reading might seem to involve fewer contentious theoretical notions than the reading I have proposed, which involves explicit talk of speech acts of assertion and denial as well as the normative notions of committive consequence and incoherence. Moreover, in the context of such a reading, we might think of the logician's non-suppositional uses of $+A$ and $-A$, expressing the truth of A and the falsity of A, as actually functioning as a means for them to *themself assert and deny*, rather than to *talk about* assertion and denial. This is the sort of interpretation Kürbis suggests on behalf of the bilateral logician.[16] Even if this is not the way the bilateral logician is *in fact* thinking of their use of signs, one might nevertheless wonder whether it is how they *should* be thinking of them.

Once again, however, the question of how the bilateralist should think about their use of a bilateral system must be understood in the context of the philosophical project in which bilateralism is being used, and, in the context of the inferentialist program, there is good reason to explicitly work, in the first instance, with normative notions rather than alethic ones. The reason is that it is a

these contents, and when he comes to the conclusion that some content must be a logical truth, then and only then does he use the judgment stroke to actually assert it (see, e.g. [3, §14, 29-31]). Now, in the *Tractatus*, Wittgenstein famously denounced this use of the judgment stroke, as "logically altogether meaningless," claiming that "it only shows that these authors hold as true the propositions marked in this way" and "belongs therefore to the propositions no more than does the number of the proposition," [29, §4.442]. Whatever the merit of Wittgenstein's criticism of the judgment stroke is, it's clear that it doesn't apply to the "assertion sign" of bilateral logic, as I've articulated its use here. Of course, we can call the signs "force-markers," but we should be clear, once again, that they are being used to *reason about* speech acts with different forces rather than being used to *perform* speech acts with different forces.

[16]This basic alethic approach to bilateralism, where the two poles of bilateralism are taken to be truth and falsity rather than assertion and denial, is defended at length by Kürbis in [14]. There, Kürbis ends up endorsing the bi-intuitionist system proposed by Wansing [27], rather than a Rumfitt-style system of the sort under consideration here, but the question of whether to articulate bilateralism in alethic terms or normative terms is independent of the specific sort of bilateral system one endorses.

basic thought of normative inferentialism (indeed, I see it as *the* basic thought motivating the development of inferentialism in Sellars and Brandom) that alethic notions can be understood as conceptually downstream from corresponding normative ones. In this way, our grasp of conceptual contents—the propositions we assert and the properties and relations we assert of things—can be explained in terms of our mastery the norms governing the use of sentences and predicates. As before, considering the case of *non*-logical, *material* contents can help make this thought particularly clear. In the case of material contents, our grasp of the fact that something's being red necessitates its being colored and excludes its being green is understood in terms of our mastery of the norms governing the use of "red," specifically, that asserting of something that it's red commits one to asserting of it that it's colored and precludes one from being entitled to assert of it (and, moreover, commits one to deny of it) that its green. It is through mastering these norms by way of linguistic training that one ultimately comes to grasp the conceptual contents expressed by "red," "colored," and so on, which can be articulated in the alethic terms I've just deployed. The same general point applies for logical contents. Our grasp on the fact that a conjunctive proposition is true just in case both conjuncts is true (and false if at least one conjunct is false) can be understood in terms of our mastery of the norms governing the making of assertions and denials in a discursive practice. Accordingly, insofar as we are giving an inferentialist theory of content (logical or material), it makes perfect sense for our theory to be couched in explicitly normative vocabulary, for only by doing so do we actually get a theory of conceptual understanding.

Of course, one might not feel the pull of the basic normative inferentialist program, and it would obviously be silly to try to independently motivate it here.[17] The point is just that this is the major philosophical program in the context of which speech act bilateralism (of both the Smiley/Rumfitt variety of concern in the present paper and the Restall/Ripley variety) has been developed and deployed. In this context, it makes perfect sense to adopt the conception of bilateralism I have put forward here, and if we adopt this conception, Kürbis's objection simply misses. There are of course other objections to bilateralism in particular or normative pragmatic theories of content and consequence in general to which I have not responded here.[18] So speech act bilateralism may yet face serious problems. Supposition, however, is not one of them.

[17]For my current best attempts at motivating it along the lines I've just suggested, see [21] and [23].

[18]For one major objection to speech act bilateralism (and, in fact, normative pragmatic accounts of consequence in general) to which I have *not* responded here, see a different paper of Kürbis [15].

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